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Ultra-low frequency waves in the magnetosphere

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Two recent investigations, both motivated by studies of Landau damping, are reported.
With the aid of observations from the N.E.R.C. network phase differences between stations have been studied. Clear results were obtained more easily than expected. East–west phase differences were rather small.
The possibility of deducing the flow of micropulsation energy across L shells from ground data is discussed and it is shown that tilts of the polarization ellipse are significant. Field line resonance is important and theoretical models are briefly discussed.

1. Introduction

This paper does not comply with the organizers' request for an account of wave particle interactions at ultra-low frequency, but the motivation for studying the two topics which are discussed does arise from theoretical studies of wave particle interactions, if that rather general expression is interpreted as Landau damping (Southwood, Dungey & Etherington 1969). The importance of the Poynting vector or energy flow is evident enough and so is its relation to sources and sinks of pulsation energy, which may involve negative and positive Landau damping respectively. The significance of the other topic, the east–west phase variation, is not obvious, but is in fact related to Landau damping, which at these frequencies comes from bounce resonance with a Doppler shift due to the gradient and curvature drifts. If the pulsation field is taken to vary with longitude ϕ like $e^{im\phi}$, as for an eigenfunction in an axially symmetric model, m must be an integer, but the real field shows a diurnal variation in amplitude so that, even if $e^{im\phi}$ is an adequate approximation for the eigenfunctions, it must be a superposition of several eigenfunctions. Nevertheless it is adequate to consider this simple minded form, because the significant question is just the order of magnitude of m. Southwood *et al.* (1969) found that bounce resonance is most interesting for large values of m (> 10 say) for two reasons. One is that negative Landau damping results from a gradient of particle intensity and the minimum gradient required varies like m^{-1}. The other is that strong Landau damping requires the energy of resonant particles to fall in the range of high intensity in the radiation belts and for low values of m one of the two resonant energies is too low and the other too high. Consequently for low m the Landau damping is always positive and weak. Recent observations described in the next section show only low values of east–west phase variation, but it will be pointed out that large values might yet be found elsewhere.

2. The east–west phase variation

Measurements of the phase difference between two observatories at the same geomagnetic latitude may be expressed as the number of degrees of phase difference per degree of geomagnetic longitude between the stations and this provides an order of magnitude for m, defined in the introduction. The only previous study (Herron 1966) known to the authors investigated mainly transient pulsations and only one continuous pulsation (pc), the latter showing an east–west wavelength in the United States of *ca.* 800 km corresponding to quite large m (*ca.* 30) and the wavelengths of the transient pulsations ranging from 1000–10000 km.

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The National Environmental Research Council has recently set up a network of magnetic observatories in the British Isles with identical rubidium vapour magnetometers having a sensitivity of 0.025 nT and recording on paper charts with a speed of 12.5 mm/min.

The data was kindly supplied by Dr W. F. Stuart and the analysis to be quoted here was performed by Dr C. A. Green at Imperial College and is to be published. Three of the N.E.R.C. observatories are near the same geomagnetic latitude: York, Stonyhurst and Valencia (Eire). They are separated by less than Herron's wavelength and preliminary comparisons of data from York and Stonyhurst showed no phase difference greater than the probable error, indicating that a greater separation was required, but also providing reassurance that a whole wavelength would not be lost by using a large separation. Analysis of data from Stonyhurst and Valencia gave very pleasing results. The spectra of the pc data used showed a narrow band around the pulsation frequency with much greater power than the background and, while there was a clear phase difference in this power band, the phase difference outside the power band was small and systematic. It was previously feared that, at low power levels, the data might be dominated by local disturbances, possibly man-made, which would have no correlation, and it is therefore very pleasing that the phase difference is far from random. It seems likely that the disturbances outside the pulsation band are basically simultaneous, such as weak impluses which propagate in the fast mode in the magnetosphere. The major part of the phase difference observed outside the pulsation band may be accounted for by timing errors, which give a phase difference proportional to frequency. A complication must also arise due to induction effects and the coastal type of induction effect must be important at Valencia. This effect involves the polarization. The north–south disturbance was used for the phase difference analysis and the coastal effect mixes in some of the east–west component. This needs investigation, but the observed phase difference varies smoothly with frequency and it is therefore possible to proceed empirically, interpolating the curve through the pulsation band and using the interpolated section as a baseline for the phase difference in the pulsation band. With hindsight it appears that phase difference analysis is feasible with dissimilar instruments and over separations of many degrees of longitude. The phase differences found for about a dozen events were only a few degrees of phase per degree of longitude, which is much lower than for Herron's only pc. It may be that high m pulsations occur elsewhere and this may be settled in the next few years. High latitudes need to be explored and the International Magnetospheric Study may provide the required data. Another possibility is that the pulsations observed at geostationary orbit (Cummings, O'Sullivan & Coleman 1969) have such high m that the ionosphere screens them from the ground. Now in 1975, ATS6 is expected to move slowly eastwards to a new station and to pass SMS1, which also has a magnetometer, on its way. This operation has a good chance of measuring east–west phase differences at the geostationary circle.

3. The effect of the ionosphere on micropulsations observed at the ground

Hughes (1974*a*) has elucidated the effect on micropulsations of currents induced in the ionosphere. This is important in providing the boundary condition at the bottom of the magnetosphere, but what is important in this paper is the relation between the disturbance observed at the ground and that which would have been observed just above the ionosphere, a convenient and more precise level being defined as that where the mass density of protons exceeds that of oxygen ions. A simple explanation of the effect starts by noting that the electrical conductivity in the troposphere is so low that any vertical current at micropulsation frequencies must be mainly displacement current. If the vertical current is significant, the vertical electric field must then be large and consequently also the horizontal electrical field and current in the ionosphere. This suggests that the vertical current in the troposphere should be small, but in the magnetosphere the transverse mode must be involved and vertical current is generally expected. Hughes' computations confirm that currents in the ionosphere adjust the wave field in such a way that the vertical current is small below the ionosphere without requiring that it be small above. With two further assumptions the simple result is obtained that the horizontal magnetic disturbance above the ionosphere is approximately given by rotating the horizontal disturbance at the ground through a right angle, right handed with respect to the geomagnetic field. The assumptions are

(i) that the scale H of horizontal variation of the pulsation is much larger than the scale height of the ionosphere;

(ii) that the disturbance field does not exponentiate upwards like $e^{z/H}$ in the magnetosphere.

It was suggested in §2 that pulsations not satisfying (i) might escape detection at the ground. It was pointed out by Southwood that (ii) should be valid so long as H is less than the radius of the Earth.

4. The Poynting vector

Since it is impossible to deduce the Poynting vector from existing measurements on satellites, the use of ground data is discussed here following Southwood (1975). The objective is to deduce whether energy is flowing inwards or outwards in the magnetosphere and the numerous ways in which errors of sign can occur will occupy most of this section. A small flow of energy along the field lines into the ionosphere is expected, but it is the flow across L shells which concerns us. If the electric field is taken to have the ideal hydromagnetic value $-\boldsymbol{v}\wedge\boldsymbol{B}$ and crossed with the magnetic field of the wave, $\boldsymbol{b}$, the double vector product identity yields one term, $(\boldsymbol{B}\cdot\boldsymbol{b})\boldsymbol{v}$, and another parallel to $\boldsymbol{B}$, which we ignore because it is along the field line. We recognize $\boldsymbol{B}\cdot\boldsymbol{b}$ as the oscillating magnetic energy density, noting the familiar common sense of the hydromagnetic equations, and can be confident that the correct sign is given by the component of $\boldsymbol{v}$ in phase with $\boldsymbol{B}\cdot\boldsymbol{b}$, which may now be viewed as the magnetic pressure which contributes to the acceleration $i\omega\boldsymbol{v}$. The magnetospheric oscillations may be pictured in terms of moving field lines and from the inception of this picture emphasis has been given to field line resonance (Dungey 1955). The fundamental period for a field line is twice the travel time for a transverse Alfvén wave between the ionospheric ends of the field line and increases with L, but also has a large sudden decrease going out through the plasmapause.

The phase relations between the two perpendicular velocity components and $\boldsymbol{B}\cdot\boldsymbol{b}$ will now be discussed by using the following idealized scenario, which will be further explained.

pulsation period < resonant period (i)

	here	$\frac{1}{4}$ wavelength downwave
now	maximum $\boldsymbol{B}\cdot\boldsymbol{b}$ (outward acceleration) (iii)	downwave acceleration (ii)
$\frac{1}{4}$ period later	(outward velocity)	maximum $\boldsymbol{B}\cdot\boldsymbol{b}$ downwave velocity
$\frac{1}{4}$ period later still		(outward velocity)

(i) This case is chosen for ease of visualization. The resonant period is derived from the restoring force due to the curving of the field line by the wave and to obtain a shorter period this force must be reinforced by the gradient of the magnetic pressure, so that the phase relation between acceleration and pressure is the familiar one as in acoustics. The consequences of reversing this assumption will be discussed later.

(ii) Downwave means east or west in the direction of phase velocity and any east–west variation of amplitude is ignored. The amplitude may have a broad maximum in local time in which case this neglect would be justified for observations near the maximum. Failing this, at least the amplitude variation should not be correlated with the phase variation.

(iii) The features in brackets are based on a separate and distinct assumption: the amplitude decreases outwards. This would be the case a little way out beyond a resonant field line, where the resonant period increased and the amplitude decreased.

Two arguments are used to develop the scenario from 'now'. One is simply the propagation of the wave: all features occur a quarter wavelength downwave a quarter period later. The other is local development in time, acceleration becoming velocity in a quarter period. Including the features in brackets the scenario predicts that outward velocity follows a quarter period after downwave velocity and hence yields the sense of rotation of the polarization ellipse for $\boldsymbol{v}$, and the sense of rotation is the same for all vectors describing the wave and can be mapped to the ground by preserving the handedness with respect to the geomagnetic field.

The relation between the value of $\boldsymbol{v}$ in or near the equatorial plane and the value of $\boldsymbol{b}$ just above the ionosphere is easily pictured in terms of moving field lines and the Hughes rotation (§ 3) relates these to $\boldsymbol{b}$ on the ground which is measured. The polarization ellipse can be measured at one observatory, but may be affected by induction currents in the oceans or lithosphere (Hughes 1974*b*). If the phase and amplitude variations in both the north–south and east–west directions are measured at least the sense of rotation of the polarization can be predicted by our scenario, so that there is redundancy providing a check. On the other hand, if only one observatory is used, but the north–south amplitude variation assumed from the argument used in (iii), the sense of rotation determines whether the phase velocity is east or west. Consider now the effect of reversing the choice (i) so that the pulsation period exceeds the resonant period. The only change needed is to replace maximum $\boldsymbol{B}\cdot\boldsymbol{b}$ by minimum $\boldsymbol{B}\cdot\boldsymbol{b}$ and, since $\boldsymbol{B}\cdot\boldsymbol{b}$ cannot be inferred from ground observations independently, the above discussion needs no modification. Unfortunately the quantity affected is our objective, the Poynting vector, and note firstly that the east–west component of the Poynting vector now becomes upwave, because the downwave velocity is in antiphase with $\boldsymbol{B}\cdot\boldsymbol{b}$.

Our real objective is the inward or outward component or outward component of the Poynting vector, which is absent in our scenario, but clearly involves a component of outward velocity in quadrature with that shown in brackets, and hence a phase variation which would be north–south on the ground. Measurements have recently begun (Fukunishi & Lanzerotti 1974). Even so it is necessary to know whether the resonant period is longer or shorter than the pulsation period and, if this is known, the polarization can be used. Assuming the resonant period is the longer we want the component of outward velocity in phase with the downwave velocity. If this component is not zero, the major axis of the polarization ellipse will be neither north–south nor east–west, but 'tilted', and the sign of the inward or outward Poynting vector depends on which quadrants contained the major axis. Note that the sign is reversed by the Hughes rotation.

5. Theoretical models of field line resonance

Apart from computations of resonant frequency on isolated magnetic shells for disturbances which are purely toroidal or poloidal (east–west or north–south magnetic polarization in the magnetosphere) (see, for example, Orr 1973) models of field line resonance structure have only been constructed in simplified plasma and magnetic field models (Southwood 1974; Chen & Hasegawa 1974; Southwood 1975). It is found (Southwood 1974) that in the presence of a source at an outer boundary energy is absorbed in the vicinity of the resonance shell. This absorption is independent of the dissipation present in the system. Southwood (1975) pictures this in terms analogous to the absorption of energy by an eigenmode of a system. Part of the disturbance in the plasma is in quadrature with the source ('the eigenmode') part in phase. The part in phase exhibits a general decrease in amplitude towards the resonance while the part in quadrature peaks there. Such a model naturally predicts a Poynting flux towards the resonance from the source and the strong energy absorption in the vicinity of the resonance suggests that the Poynting flux would be substantially reduced inward of the resonance for a source at an outer boundary.

Certainly a large class of pulsations appear to have an outer boundary source. Reported features of high latitude pulsations (Samson, Jacobs & Rostoker 1971) fit well with such a model (Southwood 1974) and in particular the observations are consistent with the high latitude waves propagating eastward in the afternoon, westward in the morning. This latter feature strongly suggests the boundary energy source is solar wind driven waves unstable due to the Kelvin–Helmholtz instability.

Now the argument of the previous section which attempts to predict energy flux on the basis of ground information on polarization needs modification in the vicinity of resonance. In this region specific account would need to be taken in going through the physical arguments of dissipative effects. If the Poynting flux is in the same direction inward (or outward) on each side of resonance the polarization ellipse changes tilt across the region as does the sense of rotation. The simple theories that have been presented to now suggest such changes in the magnetosphere take place on scale $(\gamma/\omega)l$ where l is the scale of variation across the magnetic field of the local resonance frequency and γ is the damping decrement experienced by an Alfvén wave in the absence of an energy source. The tendency of the ionosphere to shield fast horizontal variations mentioned earlier may enlarge the scale on which variations on the ground are observed. As a result this may further complicate interpretation of ground data.

Observational reports of tilts of polarization ellipse are rather scarce possibly because this can easily be affected by local induction effects. Fam Van-Chi, Yanowskiy & Kovtun (1968) report tilts at a conjugate pair of stations which show a diurnal variation of sense. Such behaviour is also reported by Lanzerotti, Fukunishi & Chen (1974). Fukunishi & Lanzerotti (1974) observe tilts at 4 stations between $L = 3.2$ and 4.4 (one station is in the southern hemisphere) and interpret their results in terms of a field line resonance model (Lanzerotti *et al.* 1974). It does seem that too many free parameters remain in the model they use for definitive interpretation.

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